Conditional probability independent variables

AboutStatisticsNumber TheoryJavaData StructuresPrecalculusCalculusSuppose one wants to know the probability that the roll of two dice resulted in a 5 if it is known that neither die showed a 1 or a 6. Note that knowing neither die showed a 1 or a 6 reduces the sample space normally associated with rolls of two dice down to: $$\begin{array}{c|c|c|c|c|c} & 2 & 3 & 4 & 5 \\\hline 2 & (2,2) & (2,3) & (2,4) & (2,5) \\\hline 3 & (3,2) & (3,3) & (3,4) & (3,5) \\\hline 4 & (4,2) & (4,3) & (4,4) & (4,5) \\\hline 5 & (5,2) & (5,3) & (5,4) & (5,5) \\\hline \end{array}$$ After shrinking the sample space, the rest of the calculation is routine. There are now only 2 cases out of 16 where the sum of the dice is 5. Thus, the probability we seek is $2/16 = 1/8$. Now, more generally, consider the task of calculating the probability of some event B under the condition that some other event A has occurred. We denote this probability by $P(B|A)$, calling the function applied a conditional probability function. When considering $P(B|A)$ we know A has occurred, which means our sample space essentially shrinks from what it previously was to just those outcomes present in A itself. Revisiting the example above using this new notation, let B be the event of rolling a sum of 5 on two dice; and let A be the event where neither die rolled shows a 1 or 6. By examining the shrunken sample space, we previously found $$P(B|A) = \frac{2}{16}$$ However, there is another way to make this calculation when $P(A) eq 0$. Consider the following: $$\frac{P(A \cap B)}{P(A)} = \frac{\displaystyle{\left(\frac{2}{36}\right)}}{\displaystyle{\left(\frac{16}{36}\right)}} = \frac{2}{16}$$ Similarly, we expect for any events A and B when $P(A) eq 0$: $$P(B | A) = \frac{P(A \cap B)}{P(A)}$$ Multiplying the left and right sides above by $P(A)$ we have the following when $P(A) eq 0$,† $$\displaystyle{P(A \cap B) = P(A) \cdot P(B|A)}$$ The above establishes the important rule shown below, which is especially useful for finding probabilities of compound events: The Multiplication Rule : $P(A \textrm{ and } B) = P(A) \cdot P(B|A)$, for any events A and B, where $P(A) eq 0$ Of course, this rule is entirely symmetric as $A \cap B$ is the same as $B \cap A$. Thus, we also can say $P(A \textrm{ and }B) = P(B) \cdot P(A|B)$. † : For completeness, note that if $P(A)=0$, we can still compute $P(A \cap B)$ -- just by different means. Recall that we have previously shown if $C \subset D$ then $P(C) \le P(D)$. Note that $C = A \cap B$ is a subset of $D = A$. Thus $P(A \cap B) \le P(A) = 0$. Since no probability is negative, it must then be the case that $P(A \cap B) = 0$. Independent Events We normally think of events A and B as independent when knowledge of one of these events occurring does not affect the probability that the other occurs. It may not be clear what this means if either A or B is a null event -- as one would never have knowledge of a null event's occurance (remember null events never occur). For the moment then, presume these two probabilities are non-zero. Under this presumption, when A and B are independent, we can write both $P(B|A) = P(B)$ and $P(A|B) = P(A)$, as both $P(B|A)$ and $P(A|B)$ are well-defined when both $P(A)$ and $P(B)$ are non-zero. Consequently, $$P(A\text{ and }B) = P(A) \cdot P(B|A) \quad \Longleftrightarrow \quad P(A\text{ and }B) = P(A) \cdot P(B)$$ Let us now turn this into the definition for independence. That is to say, Events A and B are independent means $P(A\textrm{ and }B) = P(A) \cdot P(B)$. Doing this is certainly consistent with our earlier notion of independence, but it also expands this idea now to null events. Specifically, it requires null events be independent of other events. Hopefully, one finds reasonable that the knowledge of one event's occurrence can't affect the probability of some other event that had zero probability of occurring in the first place. To see how this definition requires null events be independent of other events, suppose A is a null event and B is any other event tied to the same sample space. Certainly then, we have $P(A) = 0$. Thus, the right side of the defining equation above (i.e., $P(A) \cdot P(B)$) equals zero. It remains to show the left side must be zero as well. Again, recall that we have previously shown if $C \subset D$, then $P(C) \le P(D)$. Noting that $C = A \cap B$ is a subset of $D = A$, which has probability zero -- it must be the case that $P(A \cap B) \le 0$. As no probability is negative, we have $P(A \textrm{ and } B) = P(A \cap B) = 0$. As such, the left and right sides of the equation defining independence above agree, and A is thus independent of B! Replacement vs. No Replacement Turning our attention to a practical consequence of independence, consider the context of selecting members from a given population to decide if some characteristic is present in the members selected. Importantly, one can make this selection either with or without replacement. To see what this means, consider the following example: Suppose one intends to test some selection from a group of batteries in order to classify them as either "good" or "defective". Suppose also that this group consists of 10 batteries, and 4 of them are defective. Note that the probability of drawing one of the 4 defective batteries from the group of 10 is 4/10 = 0.40. Suppose two batteries are selected. One way in which this might happen is for the first one to be selected and tested and then returned to the group. Then a second battery is selected and tested in the same way. In this way, the probability of drawing a defective battery is identical for both selections. As the probability of drawing a defective battery on the second draw does not change from the probability of drawing a defective battery on the first draw, we note that the event of drawing a defective battery on the first draw and the event of drawing a defective battery on the second draw are hence independent. However, one might not want to allow for the possibility that the same battery is tested twice. (If the context were different, and we were selecting people to complete a survey, instead of batteries, eyebrows would certainly be raised if we allowed a person t fill out the survey twice!) To accommodate this new restriction, one could set the first battery tested aside and then draw the second battery from the remaining nine. However, note that this changes the probabilities involved. If the first battery drawn was defective, the conditional probability that the second battery will be found to be defective is now $3/9 = 1/3$. If the first battery drawn was not defective, the conditional probability the second battery will be found to be defective is instead $4/9$, which is still different from the probability of $4/10$, that the first battery was defective. Consequently, the events of drawing a defective battery on the first and second draws, respectively, are not independent. Different techniques must then be used in these two different circumstances. The former often leads to simpler calculations, but the latter is often more appropriate. Fortunately, the numerical difference between the two probabilities involved is very small when the population is large enough when compared to the sample size (i.e., the number of elements drawn from the population). As a guideline, if the sample size is no more than 5% of the population size, the selections may be treated as independent, if desired, and the results are likely to still be close enough to the truth to be useful. Conditional probability is the probability of an event occurring given that another event has already occurred. The concept is one of the quintessential concepts in probability theoryTotal Probability RuleThe Total Probability Rule (also known as the law of total probability) is a fundamental rule in statistics relating to conditional and marginal. Note that conditional probability does not state that there is always a causal relationship between the two events, as well as it does not indicate that both events occur simultaneously.The concept of conditional probability is primarily related to the Bayes' theoremBayes' TheoremThe Bayes theorem (also known as the Bayes' rule) is a mathematical formula used to determine the conditional probability of events., which is one of the most influential theories in statistics.Formula for Conditional ProbabilityWhere:P(A|B) – the conditional probability; the probability of event A occurring given that event B has already occurredP(A ∩ B) – the joint probability of events A and B; the probability that both events A and B occurP(B) – the probability of event BThe formula above is applied to the calculation of the conditional probability of events that are neither independentIndependent EventsIn statistics and probability theory, independent events are two events wherein the occurrence of one event does not affect the occurrence of another event nor mutually exclusive.Another way of calculating conditional probability is by using the Bayes' theorem. The theorem can be used to determine the conditional probability of event A, given that event B has occurred, by knowing the conditional probability of event B, given the event A has occurred, as well as the individual probabilities of events A and B. Mathematically, the Bayes' theorem can be denoted in the following way:Finally, conditional probabilities can be found using a tree diagram. In the tree diagram, the probabilities in each branch are conditional.Conditional Probability for Independent EventsTwo events are independent if the probability of the outcome of one event does not influence the probability of the outcome of another event. Due to this reason, the conditional probability of two independent events A and B is:P(A|B) = P(A)P(B|A) = P(B)Conditional Probability for Mutually Exclusive EventsIn probability theory, mutually exclusive eventsMutually Exclusive EventsIn statistics and probability theory, two events are mutually exclusive if they cannot occur at the same time. The simplest example of mutually exclusive are events that cannot occur simultaneously. In other words, if one event has already occurred, another can event cannot occur. Thus, the conditional probability of mutually exclusive events is always zero.P(A|B) = 0P(B|A) = 0Additional ResourcesCFI offers the Financial Modeling & Valuation Analyst (FMVA)™Become a Certified Financial Modeling & Valuation Analyst (FMVA)®CFI's Financial Modeling and Valuation Analyst (FMVA)® certification will help you gain the confidence you need in your finance career. Enroll today! certification program for those looking to take their careers to the next level. To keep learning and advancing your career, the following CFI resources will be helpful:ForecastingForecastingForecasting refers to the practice of predicting what will happen in the future by taking into consideration events in the past and present. Basically, it is a decision-making tool that helps businesses cope with the impact of the future's uncertainty by examining historical data and trends.Law of Large NumbersLaw of Large NumbersIn statistics and probability theory, the law of large numbers is a theorem that describes the result of repeating the same experiment a large number ofNonparametric TestsNonparametric TestsIn statistics, nonparametric tests are methods of statistical analysis that do not require a distribution to meet the required assumptions to be analyzedQuantitative AnalysisQuantitative AnalysisQuantitative analysis is the process of collecting and evaluating measurable and verifiable data such as revenues, market share, and wages in order to understand the behavior and performance of a business. In the era of data technology, quantitative analysis is considered the preferred approach to making informed decisions.

conditional probability non independent variables. conditional probability of sum of independent random variables