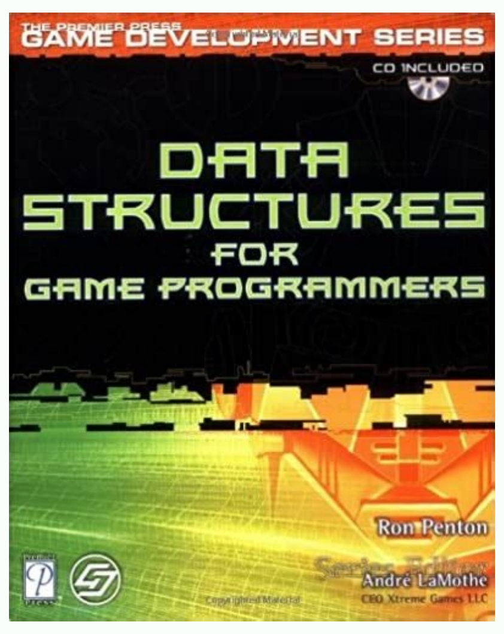
Mean value theorem practice problems and solutions pdf online book pdf format



The derivative of this function is, $$f'\left(x \right) = 20{x^4} + 3{x^2} + 7$$ Because the exponents on the first two terms are even we know that the first two terms will always be greater than or equal to zero and we are then going to add a positive number onto that and so we can see that the smallest the derivative will ever be is 7 and this contradicts the statement above that says we MUST have a number c such that $f'\left(c \right) = 0$. In most traditional textbooks this section comes before the sections containing the First and Second Derivative Tests because many of the proofs in those sections need the Mean Value Theorem. First define $A = \left({a,f\left(a \right)} \right)$ and $B = \left({b,f\left(b \right)} \right)$ and then we know from the Mean Value theorem that there is a c such that $a < c < b$ and that $$f'\left(c \right) = \frac{{f\left(b \right) - f\left(a \right)}}{{b - a}}$$ Now, if we draw in the secant line connecting A and B then we can know that the slope of the secant line is, $$\frac{{f\left(b \right) - f\left(a \right)}}{{b - a}}$$ Likewise, if we draw in the tangent line to $f\left(x \right)$ at $x = c$ we know that its slope is $f'\left(c \right)$. To do this we'll use an argument that is called contradiction proof. To see the proof see the Proofs From Derivative Applications section of the Extras chapter. In this section we want to take a look at the Mean Value Theorem. Now, take any two x's in the interval $\left({a,b} \right)$, say ${x_1}$ and ${x_2}$. $$f'\left(x \right) = 3{x^2} + 4x - 1$$ Now, to find the numbers that satisfy the conclusions of the Mean Value Theorem all we need to do is plug this into the formula given by the Mean Value Theorem. What is the largest possible value for $f\left({15} \right)$? We can see this in the following sketch. We can't say that it will have exactly one root. Now, because $f\left(x \right)$ is a polynomial we know that it is continuous everywhere and so by the Intermediate Value Theorem there is a number c such that $0 < c < 1$ and $f\left(c \right) = 0$. Before we take a look at a couple of examples let's think about a geometric interpretation of the Mean Value Theorem. To do this note that $f\left(0 \right) = - 2$ and that $f\left(1 \right) = 10$ and so we can see that $f\left(0 \right) < 0 < f\left(1 \right)$. For instance if we know that $f\left(x \right)$ is continuous and differentiable everywhere and has three roots we can then show that not only will $f'\left(x \right)$ have at least two roots but that $f''\left(x \right)$ will have at least one root. The reason for covering Rolle's Theorem is that it is needed in the proof of the Mean Value Theorem. Also note that if it weren't for the fact that we needed Rolle's Theorem to prove this we could think of Rolle's Theorem as a special case of the Mean Value Theorem. What we'll do is assume that $f\left(x \right)$ has at least two real roots. First, notice that because we are assuming the derivative exists on $\left(a,b \right)$ we know that $f\left(x \right)$ is differentiable on $\left(a,b \right)$. Example 1 Show that $f\left(x \right) = 4{x^5} + {x^3} + 7x - 2$ has exactly one real root. This fact is very easy to prove so let's do that here. But if we do this then we know from Rolle's Theorem that there must then be another number c such that $f'\left(c \right) = 0$. Let's also suppose that we know that $f\left(x \right)$ has two roots. We'll close this section out with a couple of nice facts that can be proved using the Mean Value Theorem. Now, by assumption we know that $f\left(x \right)$ is continuous and differentiable everywhere and so in particular it is continuous on $\left[{a,b} \right]$ and differentiable on $\left({a,b} \right)$. Here is the theorem. This is a problem however. Fact 1 If $f'\left(x \right) = 0$ for all x in an interval $\left({a,b} \right)$ then $f\left(x \right)$ is constant on $\left({a,b} \right)$. Before we get to the Mean Value Theorem we need to cover the following theorem. This is actually a fairly simple thing to prove. $$f\left({15} \right) - f\left(6 \right) = f'\left(c \right)\left({15 - 6} \right)$$ Plugging in for the known quantities and rewriting this a little gives, $$f\left({15} \right) = f\left(6 \right) + f'\left(c \right)\left({15 - 6} \right) = - 2 + 9f'\left(c \right)$$ Now we know that $f'\left(x \right) \le 10$ so in particular we know that $f'\left(c \right) \le 10$. Mean Value Theorem Suppose $f\left(x \right)$ is a function that satisfies both of the following. Let's take a look at a quick example that uses Rolle's Theorem. $$\begin{align*}f'\left(c \right) & = \frac{{f\left(2 \right) - f\left({ - 1} \right)}}{{2 - \left({ - 1} \right)}}\\ 3{c^2} + 4c - 1 & = \frac{{14 - 2}}{3} = \frac{{12}}{3} = 4\end{align*}$$ Now, this is just a quadratic equation, $$\begin{align*}3{c^2} + 4c - 1 & = 4\\ 3{c^2} + 4c - 5 & = 0\end{align*}$$ Using the quadratic formula on this we get, $$c = \frac{{ - 4 \pm \sqrt {16 - 4\left(3 \right)\left({ - 5} \right)} }}{6} = \frac{{ - 4 \pm \sqrt {76} }}{6}$$ So, solving gives two values of c. We have only shown that it exists. $f\left(x \right)$ is differentiable on the open interval $\left({a,b} \right)$. It is completely possible for $f'\left(x \right)$ to have more than one root. This means that the largest possible value for $f\left({15} \right)$ is 88. Example 3 Suppose that we know that $f\left(x \right)$ is continuous and differentiable on $\left[{6,15} \right]$. If we first define, $$h\left(x \right) = f\left(x \right) - g\left(x \right)$$ Then since both $f\left(x \right)$ and $g\left(x \right)$ are continuous and differentiable in the interval $\left({a,b} \right)$ then so must be $h\left(x \right)$. $$f\left(x \right) = {x^3} + 2{x^2} - x\hspace{0.25in}{\rm{on}}\hspace{0.25in}\left[{ - 1,2} \right]$$ Show Solution There isn't really a whole lot to this problem other than to notice that since $f\left(x \right)$ is a polynomial it is both continuous and differentiable (i.e. the derivative exists) on the interval given. It is possible for both of them to work. $f\left(x \right)$ is continuous on the closed interval $\left[{a,b} \right]$. We reached these contradictory statements by assuming that $f\left(x \right)$ has at least two roots. The number that we're after in this problem is, $$c = 0.7863$$ Be careful to not assume that only one of the numbers will work. Let's now take a look at a couple of examples using the Mean Value Theorem. Let's also suppose that we know that $f\left(6 \right) = - 2$ and that we know that $f'\left(x \right) \le 10$. $f\left(a \right) = f\left(b \right)$ Then there is a number c such that $a < c < b$ and $f'\left(c \right) = 0$. First let's find the derivative. To see that just assume that $f\left(a \right) = f\left(b \right)$ and then the result of the Mean Value Theorem gives the result of Rolle's Theorem. This means that we have, $$\begin{align*}h\left(x \right) & = c\\ f\left(x \right) - g\left(x \right) & = c\\ f\left(x \right) & = g\left(x \right) + c\end{align*}$$ which is what we were trying to show. This gives us the following, $$\begin{align*}f\left({15} \right) & = - 2 + 9f'\left(c \right)\\ & \le - 2 + \left(9 \right)10\\ & = 88\end{align*}$$ All we did was replace $f'\left(c \right)$ with its largest possible value. Show Solution It is important to note here that all we can say is that $f'\left(x \right)$ will have at least one root. That means that we will exclude the second one (since it isn't in the interval). First, we should show that it does have at least one real root. Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). It only tells us that there is at least one number c that will satisfy the conclusion of the theorem. So, by Fact 1 $h\left(x \right)$ must be constant on the interval. So, if you've been following the proofs from the previous two sections you've probably already read through this section. Show that $f'\left(x \right)$ must have at least one root. It is completely possible to generalize the previous example significantly. Fact 2 If $f'\left(x \right) = g'\left(x \right)$ for all x in an interval $\left({a,b} \right)$ then in this interval we have $f\left(x \right) = g\left(x \right) + c$ where c is some constant. Since we know that $f\left(x \right)$ has two roots let's suppose that they are a and b. Then there is a number c such that a < c < b and $$f'\left(c \right) = \frac{{f\left(b \right) - f\left(a \right)}}{{b - a}}$$ Or, $$f\left(b \right) - f\left(a \right) = f'\left(c \right)\left({b - a} \right)$$ Note that the Mean Value Theorem doesn't tell us what c is. Example 2 Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function. Note that in both of these facts we are assuming the functions are continuous and differentiable on the interval $\left[{a,b} \right]$. Doing this gives, $$f\left({{x_2}} \right) - f\left({{x_1}} \right) = f'\left(c \right)\left({{x_2} - {x_1}} \right)$$ where ${x_1} < c < {x_2}$. $$c = \frac{{ - 4 + \sqrt {76} }}{6} = 0.7863\hspace{0.25in}\hspace{0.25in}c = \frac{{ - 4 - \sqrt {76} }}{6} = - 2.1196$$ Notice that only one of these is actually in the interval given in the problem. Again, it is important to note that we don't have a value of c. Show Solution Let's start with the conclusion of the Mean Value Theorem. Example 4 Suppose that we know that $f\left(x \right)$ is continuous and differentiable everywhere. Therefore, the derivative of $h\left(x \right)$ is, $$h'\left(x \right) = f'\left(x \right) - g'\left(x \right)$$ However, by assumption $f'\left(x \right) = g'\left(x \right)$ for all x in an interval $\left({a,b} \right)$ and so we must have that $h'\left(x \right) = 0$ for all x in an interval $\left({a,b} \right)$. In addition, we know that if a function is differentiable on an interval then it is also continuous on that interval and so $f\left(x \right)$ will also be continuous on $\left(a,b \right)$. So don't confuse this problem with the first one we worked. Then since $f\left(x \right)$ is continuous and differentiable on $\left({a,b} \right)$ it must also be continuous and differentiable on $\left[{{x_1},{x_2}} \right]$. If your device is not in landscape mode many of the equations will run off the side of your device (should be able to scroll to see them) and some of the menu items will be cut off due to the narrow screen width. We'll leave it to you to verify this, but the ideas involved are identical to those in the previous example. This fact is a direct result of the previous fact and is also easy to prove. In other words $f\left(x \right)$ has at least one real root. Due to the nature of the mathematics on this site it is best views in landscape mode. Show Solution From basic Algebra principles we know that since $f\left(x \right)$ is a 5th degree polynomial it will have five roots. We also haven't said anything about c being the only root. This means that we can apply the Mean Value Theorem for these two values of x. Or, in other words $f\left(x \right)$ has a critical point in $\left({a,b} \right)$. Therefore, by the Mean Value Theorem there is a number c that is between a and b (this isn't needed for this problem, but it's true so it should be pointed out) and that, $$f'\left(c \right) = \frac{{f\left(b \right) - f\left(a \right)}}{{b - a}}$$ But we now need to recall that a and b are roots of $f\left(x \right)$ and so this is, $$f'\left(c \right) = \frac{{0 - 0}}{{b - a}} = 0$$ Or, $f'\left(x \right)$ has a root at $x = c$. We now need to show that this is in fact the only real root. To see the proof of Rolle's Theorem see the Proofs From Derivative Applications section of the Extras chapter. But by assumption $f'\left(x \right) = 0$ for all x in an interval $\left({a,b} \right)$ and so in particular we must have, $$f'\left(c \right) = 0$$ Putting this into the equation above gives, $$f\left({{x_2}} \right) - f\left({{x_1}} \right) = 0\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}f\left({{x_2}} \right) = f\left({{x_1}} \right)$$ Now, since ${x_1}$ and ${x_2}$ were any two values of x in the interval $\left({a,b} \right)$ we can see that we must have $f\left({{x_2}} \right) = f\left({{x_1}} \right)$ for all ${x_1}$ and ${x_2}$ in the interval and this is exactly what it means for a function to be constant on the interval and so we've proven the fact. This means that we can find real numbers a and b (there might be more, but all we need for this particular argument is two) such that $f\left(a \right) = f\left(b \right) = 0$. What the Mean Value Theorem tells us is that these two slopes must be equal or in other words the secant line connecting A and B and the tangent line at $x = c$ must be parallel. What we're being asked to prove here is that only one of those 5 is a real number and the other 4 must be complex roots. However, we feel that from a logical point of view it's better to put the Shape of a Graph sections right after the absolute extrema section. Rolle's Theorem Suppose $f\left(x \right)$ is a function that satisfies all of the following. Since this assumption leads to a contradiction the assumption must be false and so we can only have a single real root.

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